Math 275 Formula Sheet

Vector Formulas

Magnitude of the vector $\mathbf{v} = \langle v_1, v_2, \dots, v_n \rangle$

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

Angle between vectors

$$\cos\theta = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$$

Projection of **u** onto **v**

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$$

Cross product

$$\mathbf{u}\times\mathbf{v} = (|\mathbf{u}||\mathbf{v}|\sin\theta)\mathbf{n}$$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Box product

$$(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Lines and Planes in space

Distance from a point S to a line through P, parallel to **v**

$$d = \frac{|\overrightarrow{\mathrm{PS}}\times\mathbf{v}|}{|\mathbf{v}|}$$

Distance from a point S to a plane with normal **n**

$$d = \left|\overrightarrow{\mathrm{PS}}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Motion in Space

Position vector

$$\mathbf{r}(\mathrm{t}) = \langle x, y, z \rangle = \langle f(t), g(t), h(t) \rangle$$

Velocity vector and tangent vector

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \qquad \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

Normal vector

$$\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$$

Binormal vector

$$\mathbf{B} = \mathbf{T}\times\mathbf{N}$$

Curvature

$$\kappa = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$$

Torsion

$$\tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N} = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\mathbf{v}\times\mathbf{a}|^2}$$

Tangential and normal scalar components of acceleration

$$\mathbf{a} = a_{\mathrm{T}}\mathbf{T} + a_{\mathrm{N}}\mathbf{N}$$

$$a_{\mathrm{T}} = \frac{d}{dt}|\mathbf{v}| \quad \text{and} \quad a_{\mathrm{N}} = \sqrt{|\mathbf{a}|^2 - a_{\mathrm{T}}{}^2}$$

Length of a curve

$$L = \int_a^b |\mathbf{v}|\,dt$$

Arc length parameter with base point $P(t_0)$

$$s(t) = \int_{t_0}^{t} |\mathbf{v}(\tau)|\,d\tau$$

Functions of Several Variables

Implicit Differentiation: If $F(x,y)=0$ defines y as a differentiable function of x, then at any point where $F_y \neq 0$,

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Directional derivative at a point P_0

$$(D_{\mathbf{u}}f)_{P_0} = \left(\frac{df}{ds}\right)_{\mathbf{u},\mathrm{P}_0} = \nabla f(P_0) \cdot \mathbf{u}$$

Tangent plane to $f(x,y,z)=c$ at $P_0(x_0,y_0,z_0)$

$$f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0) + f_z(P_0)(z-z_0) = 0$$

Linearization of a function $f(x,y)$ at $P_0(x_0,y_0)$

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

First Derivative Test
If $f(x,y)$ has a local max or min at an interior point (a,b) and if the first partial derivatives exist there, then

$$f_x(a,b) = 0 \text{ and } f_y(a,b) = 0$$

Second Derivative Test

If $f_x(a,b) = f_y(a,b) = 0$, then $f(a,b)$ is a:

1) Local maximum if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$
2) Local minimum if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}^2 > 0$
3) Saddle point if $f_{xx}f_{yy} - f_{xy}^2 < 0$
4) The test is inconclusive if $f_{xx}f_{yy} - f_{xy}^2 = 0$

Lagrange Multipliers
To find the extreme values of f subject to the constraint $g=0$, solve the system of equations given by $\nabla f = \lambda \nabla g$ and $g=0$.

Multiple Integrals

Double Integral in Polar Form

$$\iint_R f(r,\theta)\, r\, dr\, d\theta$$

Triple Integral in Cylindrical Form

$$\iiint_R f\, dz\, r\, dr\, d\theta$$

Triple Integral in Spherical Form

$$\iiint_R f\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Mass

$$M = \iiint_R \delta\, dV$$

First Moments

$$M_{xy} = \iiint z\, \delta\, dV \qquad M_{xz} = \iiint y\, \delta\, dV$$

$$M_{yz} = \iiint x\, \delta\, dV$$

Center of Mass (3D)

$$\bar{x} = \frac{M_{yz}}{M} \qquad \bar{y} = \frac{M_{xz}}{M} \qquad \bar{z} = \frac{M_{xy}}{M}$$

Inertia where r is the distance to axis of rotation

$$\iint_R r^2\, \delta\, dA \qquad \iiint_R r^2\, \delta\, dV$$

Inertia about the origin (2D)

$$I_0 = I_x + I_y$$

Integration in Vector Fields

Line Integrals

$$\int_C f(x,y,z)\,ds = \int_a^b f(g(t),h(t),k(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt$$

Line Integral over a vector field (work, flow, and circulation)

$$\int_C \mathbf{F}\cdot\mathbf{T}\;ds = \int_C \mathbf{F}\cdot\frac{d\mathbf{r}}{ds}ds = \int_C \mathbf{F}\cdot d\mathbf{r}$$

$$= \int_a^b \mathbf{F}\big(\mathbf{r}(t)\big)\cdot\frac{d\mathbf{r}}{dt}dt = \int_a^b \left(M\frac{dx}{dt}+N\frac{dy}{dt}+P\frac{dz}{dt}\right)dt = \int_C Mdx+Ndy+Pdz$$

Circulation and Flux

Green's Theorem (Tangential Form, Circulation around piecewise smooth, simple closed curve C in the plane)

$$\oint_C \mathbf{F}\cdot\mathbf{T}\,ds = \oint_C Mdx+N\,dy = \iint_R (N_x-M_y)\,dxdy$$

Green's Theorem (Normal Form, Flux across piecewise smooth, simple closed curve C in the plane)

$$\oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \oint_C Mdy-N\,dx = \iint_R (M_x+N_y)\,dxdy$$

Stokes' Theorem (Circulation around piecewise smooth boundary curve C around the piecewise smooth surface S in space)

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$$

Divergence Theorem (Flux across a piecewise smooth surface S in space)

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \nabla\cdot\mathbf{F}\,dV$$

Surface Integrals

$$\iint_S G(x,y,z)d\sigma$$

Explicit Form

$$\iint_S G\big(x,y,f(x,y)\big)\sqrt{{f_x}^2+{f_y}^2+1}dA$$

Implicit Form

$$\iint_S G(x,y,z)\frac{|\nabla F|}{|\nabla F\cdot\mathbf{p}|}\,dA$$

Parametric Form

$$\iint_S G\big(f(u,v)\big),g(u,v),h(u,v))|\mathbf{r}_u\times\mathbf{r}_v|\,dA$$