

Math 175 Common Formula Sheet

Volume Formulas

Volume of a solid with cross section $A(x)$

$$V = \int_a^b A(x) dx$$

Volume by disk, revolved around x-axis

$$V = \int_a^b \pi[R(x)]^2 dx$$

Volume by washers, revolved around x-axis

$$V = \int_a^b \pi[R(x)]^2 - \pi[r(x)]^2 dx$$

Volume by cylindrical shells, revolved about vertical line

$$V = \int_a^b 2\pi \left(\begin{array}{c} \text{shell} \\ \text{radius} \end{array} \right) \left(\begin{array}{c} \text{shell} \\ \text{height} \end{array} \right) dx$$

Pappus's Theorem for Volumes (plane region revolved once about a line in the plane)

$$V = 2\pi\rho A$$

Moments and Centers of Mass

Moment about the x-axis

$$M_x = \int \tilde{y} dm$$

Moment about the y-axis

$$M_y = \int \tilde{x} dm$$

Mass

$$M = \int dm$$

Center of mass bounded by curves with constant density

$$\bar{x} = \frac{1}{M} \int_a^b \delta x [f(x) - g(x)] dx$$

$$\bar{y} = \frac{1}{M} \int_a^b \frac{\delta}{2} [(f(x))^2 - (g(x))^2] dx$$

Arc Length

Length of the curve given by $y = f(x)$, from a to b

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Length of curve defined parametrically

$$L = \int_a^b \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2} dt$$

Work

Work done by a variable force $F(x)$, along x-axis

$$W = \int_a^b F(x) dx$$

Techniques of integration

Integration by parts

$$\int u dv = uv - \int v du$$

Trigonometric Substitution

$$\text{for } \sqrt{a^2 - x^2} \text{ use } x = a \sin \theta$$

$$\text{for } \sqrt{a^2 + x^2} \text{ use } x = a \tan \theta$$

$$\text{for } \sqrt{x^2 - a^2} \text{ use } x = a \sec \theta$$

Surface Area

Area of a surface, revolved around x-axis

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$

Pappus's Theorem for Surface Areas (plane region revolved once about a line in the plane)

$$S = 2\pi\rho L$$

Integration Formulas

(Constants of integration have been omitted)

$$\int x^n dx = \frac{x^{n+1}}{n+1} \text{ for } n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int e^x dx = e^x$$

$$\int a^x dx = \frac{a^x}{\ln(a)} \text{ for } a \neq -1$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\int \sec^2 x dx = \tan x$$

$$\int \csc^2 x dx = -\cot x$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \csc x \cot x dx = -\csc x$$

$$\int \sec x dx = \ln|\sec x + \tan x|$$

$$\int \csc x dx = \ln|\csc x - \cot x|$$

$$\int \tan x dx = \ln|\sec x|$$

$$\int \cot x dx = \ln|\sin x|$$

Approximating Integrals

Trapezoidal Rule

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$$

Simpson's Rule

$$\int_a^b f(x) dx \approx \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Differential Equations

Separable differential equation

Given:

$$\frac{dy}{dx} = f(x)g(y)$$

We can separate and integrate

$$\int \frac{1}{g(y)} dy = \int f(x) dx$$

First order linear differential equation

Given:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

The solution is

$$y = \frac{1}{v(x)} \int v(x)Q(x) dx$$

where

$$v(x) = e^{\int P(x) dx}$$

Polar Coordinate Calculus

Derivative

$$\frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos(\theta) - f(\theta) \sin \theta}$$

Area under curve

$$A = \int_{\alpha}^{\beta} \frac{1}{2} [f(\theta)]^2 d\theta$$

Arc Length

$$L = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta$$

Series

The ratio test

If:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$$

Then the series is absolutely convergent

The root test

If:

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L < 1$$

Then the series is absolutely convergent

Common Series

Function	Series	Interval of convergence
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$	$(-1, 1)$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$(-\infty, \infty)$
$\sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$(-\infty, \infty)$
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$(-\infty, \infty)$
$\ln(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$(-1, 1]$
$\tan^{-1} x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$[-1, 1]$
$(1+x)^m$	$\sum_{n=1}^{\infty} \binom{m}{n} x^n = 1 + mx + \frac{m(m-1)x^2}{2!} + \dots$	$(-1, 1)$